

Design Theory and Aerodynamics of Flying Wings and Tailless Aircraft

A learning module for undergraduate aerospace engineering

This module develops the aerodynamic theory behind flying wings and tailless aircraft. It assumes a working knowledge of multivariable calculus and of low-speed aerodynamics: lift, drag, pitching moment, airfoil polars and the basics of finite-wing theory. The module covers the reasons for choosing a tailless configuration, the fundamental definitions of centre of pressure, aerodynamic centre and neutral point, longitudinal static stability, spanwise lift distributions, and the geometric design choices of twist, taper and stall control. Dynamic stability, lateral-directional modes and a full treatment of stability derivatives are deliberately left for a follow-on module.

Notation and Conventions

Throughout the module, pitching moments and angles follow the standard aircraft sign convention. A nose-up moment is positive. An aircraft is statically stable in pitch if a nose-up disturbance produces a nose-down restoring moment, which in coefficient form means that the derivative of the pitching-moment coefficient with respect to angle of attack is negative. Chordwise positions are given as fractions of the chord, measured aft from the leading edge. The symbols used are collected in the table below.

Symbol	Meaning
α	Angle of attack
C_L, C_D	Lift and drag coefficients
$C_{L,\alpha}$	Lift-curve slope, $dC_L/d\alpha$
C_m	Pitching-moment coefficient (positive nose-up)
$C_{m,ac}$	Pitching-moment coefficient about the aerodynamic centre
$C_{m,\alpha}$	Pitching-moment slope, $dC_m/d\alpha$, taken about the centre of gravity
$h_{cg}, h_{ac}, h_{np}, h_{cp}$	Chordwise positions of the centre of gravity, aerodynamic centre, neutral point and centre of pressure
SM	Static margin, $h_{np} - h_{cg}$
Λ	Sweep angle
λ	Taper ratio, tip chord divided by root chord
ε_t	Geometric twist of the tip relative to the root (negative means washout)
η	Non-dimensional semispan station, $y/(b/2)$
$\Gamma(\eta)$	Spanwise circulation distribution

1. Introduction to the Tailless Configuration

1.1 What is a flying wing?

A flying wing is an aircraft in which the lifting surface is, to a first approximation, the whole aircraft: payload, crew, fuel and systems are all housed within or blended into the wing, and there is no separate fuselage and no tail. Most engineers think immediately of the Northrop B-2 Spirit bomber, and the other well-known examples are the B-21 Raider, the X-47B unmanned combat vehicle and the gliders of the Horten brothers. The broader class of tailless aircraft includes any aircraft without a horizontal stabiliser, whether or not a fuselage or a vertical fin is retained. Flying wings are usually recognisable by their high sweep, their taper and their low profile.

The defining design consequence is that pitch and yaw stability cannot be obtained from a tail surface placed on a long lever arm behind the centre of gravity. Stability must instead be generated aerodynamically by the wing itself, through the choice of airfoil, sweep, twist and planform, or it must be supplied artificially by moving control surfaces such as elevons (combined elevator and aileron surfaces). Almost every topic in this module follows from this single constraint.

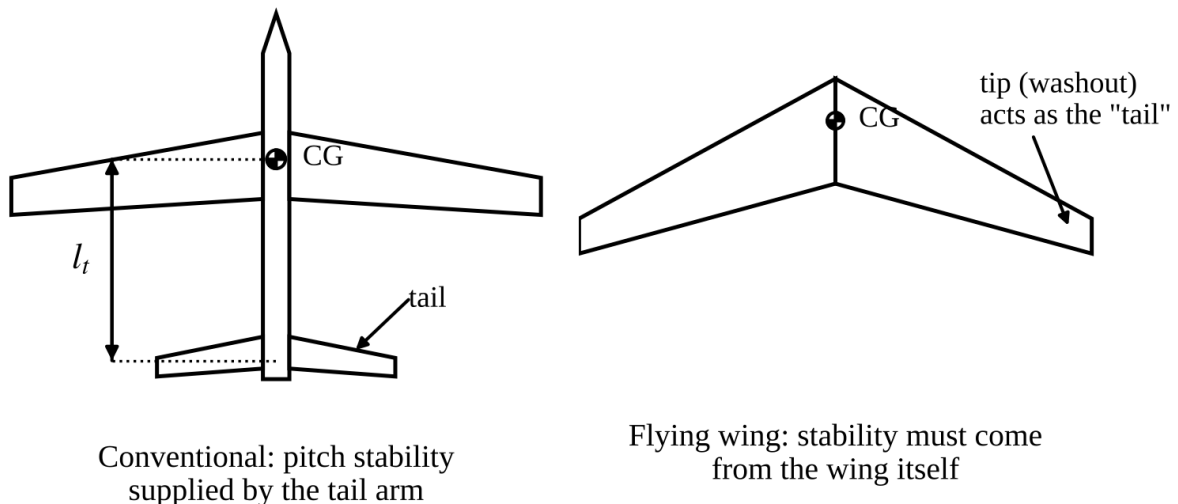


Figure 1. A conventional aircraft obtains pitch stability from the tail moment arm l_t measured from the centre of gravity (CG). A flying wing has no such arm, so the stabilising function must be built into the wing, for example through washout of a swept tip.

1.2 Benefits

The first benefit is reduced drag. A pure flying wing is theoretically the lowest-drag configuration for a fixed-wing aircraft. Without a fuselage or tail surfaces, the wetted area is smaller, and so the viscous drag (skin friction and form drag) and the interference drag between components are reduced. The lift-to-drag ratio, C_L/C_D , is therefore higher than that of the same aircraft with a tail. Higher aerodynamic efficiency means less power or fuel to cover a given distance. For a jet aircraft this is expressed by the Breguet range equation,

$$R = (V / c_T) (L/D) \ln(W_0 / W_1)$$

where R is range, V is flight speed, c_T is thrust-specific fuel consumption, and W_0 and W_1 are the initial and final weights.

The second benefit is reduced structural weight. Deleting the tail assembly removes its weight directly. In addition, a flying wing spreads its weight along the span in the same way that it spreads its lift, which lowers the net bending moment at the wing root and so reduces the minimum structural weight of spars and skins needed to maintain structural integrity in flight. This span-loading argument returns in Section 4, where it motivates the bell-shaped lift distribution.

The third benefit is stealth. With no vertical surfaces, and with no right-angle corners between fuselage, fin and stabilisers to act as corner reflectors, the radar cross-section is very low. Planform edges can also be aligned to a small number of sweep angles so that reflected radar energy is concentrated into a few narrow directions away from the radar. This is why the stealthiest aircraft in service, including the B-2, the B-21, the X-47B and newer designs, are tailless flying wings.

1.3 Challenges

If flying wings are so efficient, why are they not everywhere? The answer is that efficiency is not the only design criterion, and the aircraft must also be practical. A commercial airliner illustrates this. The best shape for a pressure vessel at high altitude is a tube or cylinder, and a wide, flat wing is a poor pressure vessel by comparison. Passengers seated far from the centreline would also experience large vertical motion whenever the aircraft rolled into a bank. A second difficulty is that a flying wing is hard to modify or upgrade. A conventional aircraft has separately defined components, such as wings, winglets, fuselage and tails, that can each be modified individually, whereas a flying wing is one continuous contoured body in which a change to one dimension changes everything.

The aerodynamic challenges are more fundamental. Flying wings often need special airfoils, or wing twist, or a combination of both, and these create trim drag and can sacrifice lift in exchange for stability. Because there are no horizontal or vertical stabilisers, pitch and yaw stability must be achieved aerodynamically or forced through control-surface actuation, rather than being provided geometrically by a tail far from the centre of gravity. Control and stability characteristics differ from those of conventional aircraft, so designing for stable, controllable take-off and landing is difficult. It is also difficult to decouple yaw, roll and Dutch roll motions, or roll and pitch motions, in tailless aircraft, especially those without a fin.

2. Fundamental Aerodynamics: Definitions

2.1 Resultant force and the three key reference points

The distribution of pressure and shear stress over an airfoil can be replaced by a single resultant force R plus a moment about a chosen reference point. In the frame of the wing, R is resolved into a normal force N and an axial force A . Rotating through the angle of attack α into the wind frame gives the lift L , perpendicular to the free stream, and the drag D , parallel to it:

$$L = N \cos \alpha - A \sin \alpha$$

$$D = N \sin \alpha + A \cos \alpha$$

The vector sum of the lift and drag is the resultant R . Three reference points are needed to discuss stability, and they differ in how the moment about them behaves.

2.2 Centre of pressure

The centre of pressure (CP) is the point along the chord at which the resultant of the pressure field acts, so that the moment about this point is zero. The pressure field changes as the angle of attack changes, so the CP location depends on α . It is a dynamic quantity. Taking moments about a reference point at chordwise position h , in the thin-airfoil, small-angle approximation,

$$C_{m,h} = C_{m,ac} + C_L (h - h_{ac})$$

Setting this moment to zero gives the CP location,

$$h_{cp} = h_{ac} - C_{m,ac} / C_L$$

This is a key result. When $C_{m,ac}$ is constant, as it is for an ideal thin airfoil, the CP moves with the lift coefficient, and the direction of its motion is determined entirely by the sign of $C_{m,ac}$.

2.3 Aerodynamic centre

The aerodynamic centre (AC) is the point along the chord about which the pitching moment is independent of angle of attack. Thin-airfoil theory places it at approximately the quarter-chord point for most subsonic airfoils. Unlike the CP, the location of the AC does not depend on α , which makes it the natural reference point for describing an airfoil: the moment about it is simply a constant, $C_{m,ac}$, throughout the linear range.

2.4 Neutral point and static margin

The neutral point (NP) is the centre-of-gravity location at which the aircraft is neutrally stable, that is, where the pitching-moment slope about the centre of gravity is zero. For an isolated airfoil or wing, the neutral point and the aerodynamic centre are synonymous. For a complete conventional aircraft, the neutral point is the aerodynamic centre of the whole configuration, which is shifted rearward from the wing's aerodynamic centre by the tail:

$$h_{np} = h_{ac,wing} + V_H (a_t / a) (1 - d\epsilon/d\alpha) \eta_t - (\text{fuselage terms})$$

Here V_H is the tail volume coefficient, a_t and a are the lift-curve slopes of the tail and the wing, $d\epsilon/d\alpha$ is the downwash gradient at the tail, and η_t is the tail efficiency. For a tailless aircraft all of the tail terms vanish, and the neutral point reduces to the aerodynamic centre of the wing itself (with a planform-dependent correction for swept wings). The distance between the neutral point and the centre of gravity is the static margin,

$$SM = h_{np} - h_{cg}$$

A positive static margin means that the centre of gravity is ahead of the neutral point, which is the stable arrangement. A zero static margin is neutral stability, and a negative value is unstable.

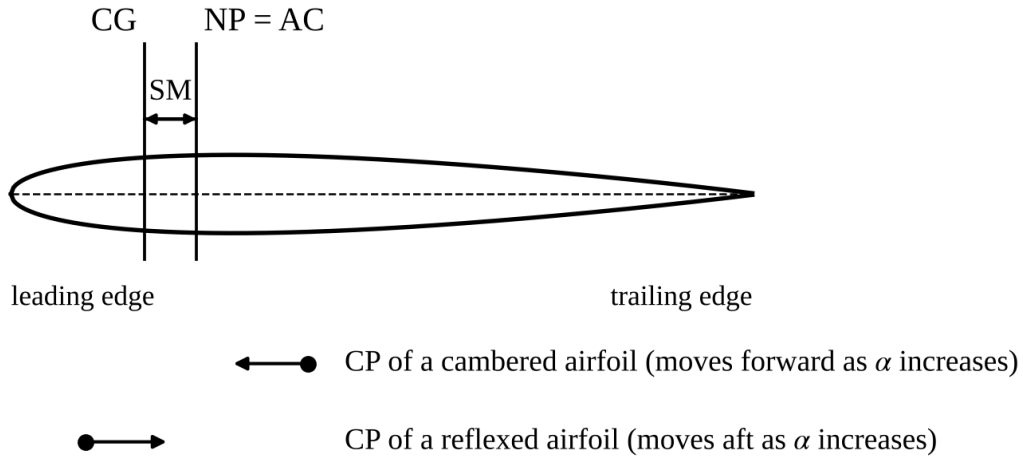


Figure 2. Chordwise stations on a section. The centre of gravity lies ahead of the neutral point (equal to the aerodynamic centre for a tailless wing) by the static margin SM. The centre of pressure of a cambered airfoil moves forward as the angle of attack increases, whereas that of a reflexed airfoil moves aft.

3. Longitudinal Static Stability

3.1 The stability condition

Consider a lifting wing with positive lift coefficient. By the definitions of Section 2, the pitching-moment coefficient about the centre of gravity is

$$C_{m,cg} = C_{m,ac} + C_L (h_{cg} - h_{ac})$$

For small angles of attack, the lift coefficient is proportional to α , so differentiating with respect to either variable gives the same sign. Differentiating with respect to C_L , with $h_{ac} = h_{np}$ for the wing alone,

$$dC_{m,cg} / dC_L = h_{cg} - h_{np} = -SM$$

and, with respect to angle of attack,

$$C_{m,\alpha} = C_{L,\alpha} (h_{cg} - h_{np}) = -C_{L,\alpha} SM$$

Static longitudinal stability requires the slope $C_{m,\alpha}$ to be negative. Because $C_{L,\alpha}$ is positive, this requires $h_{cg} < h_{np}$: the centre of gravity must be forward of the neutral point. Graphically, this is a negative slope on a plot of pitching moment against angle of attack. This is the version of the neutral-point relationship for longitudinal stability with the tail terms removed. It tells us that the centre of gravity must be ahead of the aerodynamic centre of the wing for flight, but it does not by itself guarantee stable gliding flight, because the wing must also be able to trim, that is, to reach $C_{m,cg} = 0$, at a useful positive lift coefficient. That is the subject of Section 3.3.

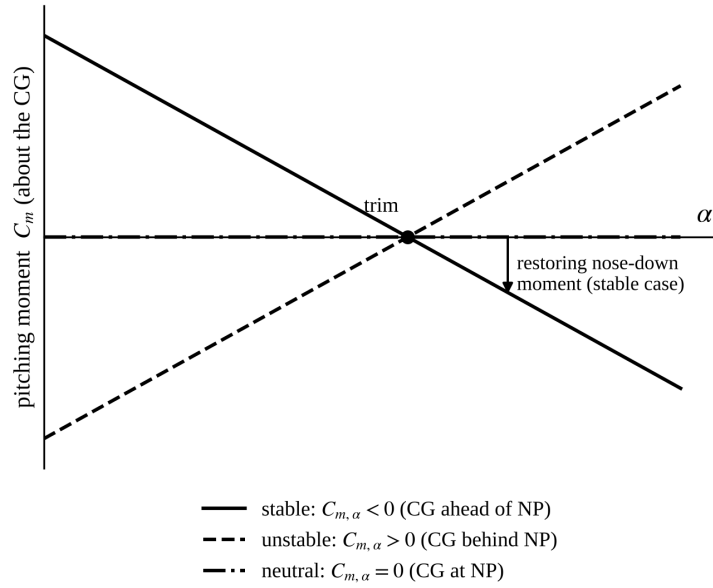


Figure 3. Pitching moment about the CG as a function of angle of attack for three CG positions. All three lines pass through the same trim point. Only the negative-slope line returns the aircraft toward trim after a disturbance.

3.2 Stability viewed through centre-of-pressure travel

A second, more physical form of the stability condition links stability to the way the centre of pressure moves as the angle of attack changes. The moment about the centre of gravity can be written as the lift acting at the CP,

$$C_{m,cg} = -C_L (h_{cp} - h_{cg})$$

and differentiating with the product rule gives

$$C_{m,\alpha} = -C_{L,\alpha} (h_{cp} - h_{cg}) - C_L dh_{cp}/d\alpha$$

Now consider the trimmed condition, in which $C_{m,cg} = 0$. The CP must then coincide with the centre of gravity, so the first term vanishes and what remains is

$$C_{m,\alpha} = -C_L dh_{cp}/d\alpha$$

For positive lift, stability therefore depends on the sign of $dh_{cp}/d\alpha$, and the margin of stability depends on its magnitude. The stability condition is that the centre of pressure must move aft as the angle of attack increases. Differentiating the CP expression of Section 2.2 gives its rate of travel,

$$dh_{cp}/d\alpha = C_{m,ac} C_{L,\alpha} / C_L^2$$

so the direction of CP travel is fixed by the sign of $C_{m,ac}$. A positively cambered, high-lift airfoil has a negative $C_{m,ac}$, which means that $dh_{cp}/d\alpha$ is negative and the CP moves forward as α increases. Physically, an increase in lift then acts further forward and adds nose-up moment, which amplifies the disturbance. This is exactly why wings built from positively cambered airfoils require a tail plane for stability. A symmetric airfoil has $C_{m,ac} = 0$, so its CP is fixed at the aerodynamic centre. An airfoil with positive $C_{m,ac}$, such as a reflexed airfoil, has a CP that moves aft with α , and this is the behaviour a tailless wing needs.

As a consistency check, combining $C_{m,\alpha} = -C_L dh_{cp}/d\alpha$ with the expression above gives $C_{m,\alpha} = -C_{m,ac} C_{L,\alpha} / C_L$, and using the trim condition of the next section, $C_{m,ac} = C_L SM$, this reduces to $C_{m,\alpha}$

$= -C_{L,\alpha}$ SM, which is Section 3.1 again.

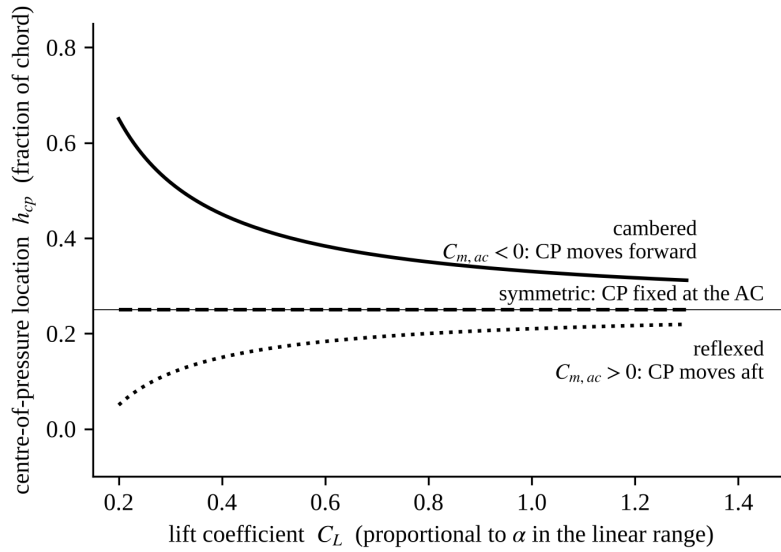


Figure 4. Centre-of-pressure location $h_{cp} = h_{ac} - C_{m,ac}/C_L$ for three airfoil types, with an aerodynamic centre at 0.25 chord. The values of $C_{m,ac}$ are illustrative (-0.08 , 0 and $+0.04$). At high lift every curve approaches the aerodynamic centre.

3.3 Trimming a tailless wing: the core design equation

For a given centre of gravity, the aircraft is in trim when the moment about the centre of gravity is zero at the design lift coefficient. For a tailless wing, with $h_{ac} = h_{np}$, the moment equation of Section 3.1 gives

$$C_{m,cg} = C_{m,ac} - C_L \text{ SM} = 0$$

which can be rearranged to

$$C_{m,ac} = C_{L,\text{design}} \text{ SM}$$

This is the central requirement of tailless design. A stable, trimmed flying wing needs a net positive, nose-up, pitching moment about its aerodynamic centre, equal to the design lift coefficient multiplied by the static margin. For example, a wing with a static margin of 0.08 that is to trim at $C_L = 0.5$ needs $C_{m,ac} = +0.04$ for the complete wing.

Compare this with an ordinary positively cambered wing, whose $C_{m,ac}$ is negative and gives a nose-heavy moment. Without a tail, the only way to balance a nose-heavy wing at zero angle of attack is to put the centre of gravity behind the aerodynamic centre. That arrangement can be balanced in a static condition, but it is unstable to any pitch disturbance, since then $C_{m,\alpha}$ is positive. Likewise, a symmetric airfoil with the centre of gravity exactly at the aerodynamic centre has zero pitching moment at $\alpha = 0$, but it also has zero lift there and no stability margin. The wing must therefore generate its own stabilising moment. There are two common ways to do this, and they are treated in the next two subsections.

3.4 Approach one: the reflexed airfoil

A reflexed airfoil is one whose trailing edge is bent slightly upward, so that the mean camber line has an S shape. The upturned trailing edge carries a reduced lift, or a download, behind the quarter chord, which produces a positive, nose-up pitching moment about the aerodynamic centre. The resulting airfoil has $C_{m,ac} > 0$, a positive zero-lift pitching moment, and a centre of pressure that moves aft as α increases. With the centre of gravity placed forward of the aerodynamic centre, such a wing can both trim at positive lift and be stable to pitch disturbances. A reflexed section also retains a positively cambered leading-edge region, so it offers better low-speed characteristics than a symmetric airfoil in the same flying-wing application.

The alternative ways to cancel the destabilising CP travel of a cambered airfoil are to add a tail plane aft of the centre of gravity, or a canard forward of it. Neither is available here, because the aim is a tailless aircraft, so the stabilising moment must be generated by the wing itself.

To make the effect concrete, a linear vortex-lattice analysis of a rectangular wing of unit chord can be run for three sections: a symmetric one, a positively cambered one (such as the NACA 4415) and a reflexed one (such as the MH 61). Figure 5 shows the form of the results, using illustrative numbers and a centre of gravity placed ahead of the aerodynamic centre so that all three lines have a negative slope. The slope of the moment curves confirms that all three wings have their neutral point at the quarter-chord, as thin-airfoil theory predicts. The symmetric wing has zero moment at zero angle of attack. The cambered wing reaches zero moment only at negative lift, and its moment at zero lift is negative, or nose-heavy. The reflexed wing has a positive zero-lift moment and reaches zero moment at positive lift, so it can trim in useful flight. Finally, when the centre of pressure is plotted against α , it moves forward for the cambered section and aft for the reflexed section, with singularities at the angles at which the lift, and so the moment, passes through zero.

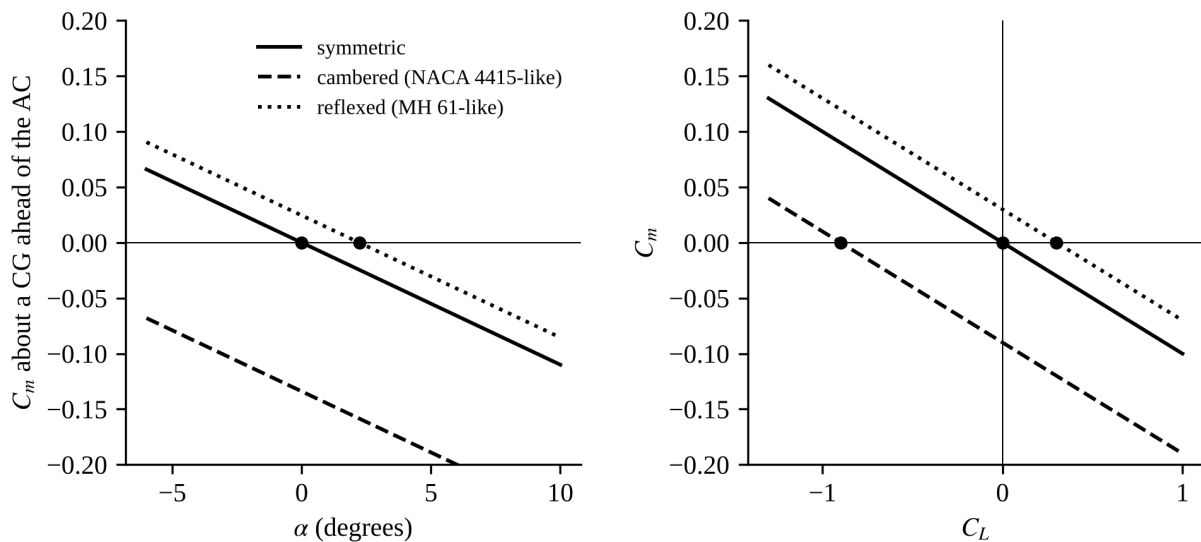


Figure 5. Illustrative moment curves about a CG that lies 0.10 chord ahead of the aerodynamic centre. The dots mark zero-moment (trim) points. The cambered section trims only at negative lift; the reflexed section trims at positive lift. Left: against angle of attack. Right: against lift coefficient.

Reflex comes at a price, and this is why not every aircraft uses it. The reflexed trailing edge carries a download that balances some of the lift produced by the front of the airfoil, and the overall loss of lift can be anywhere from about 9 to 16 percent compared with an identical airfoil without reflex. Reflexed airfoils are also reported to have a slightly higher profile drag. The lesson is that pitching stability

cannot be obtained without a loss of lift.

3.5 Approach two: wing twist

The second approach is to build a small amount of tail into the wing itself, in the form of wing twist, or washout. A swept wing whose tip is set at a lower incidence than its root carries less lift, or even a download, at the tip, and the tip sits far aft of the centre of gravity. The resulting nose-up moment has the same effect as the download on a conventional tail. This approach is developed in Section 5.

4. Lift Distributions

4.1 Background from lifting-line theory

In Prandtl's lifting-line theory a finite wing sheds a sheet of trailing vortices, and this sheet induces a vertical velocity, the downwash w , at the wing. The downwash tilts the local flow, and with it the local lift vector, by a small induced angle $\alpha_i = -w/V$ (with w measured downwards, so that α_i is negative). Because the lift vector is tilted rearward, it has a component along the flight direction that acts as drag, the induced drag. The circulation distribution $\Gamma(y)$ determines the lift per unit span, $L'(y) = \rho V \Gamma(y)$, and, through the vortex sheet, it also determines the downwash. Everything that follows is a consequence of choosing Γ .

4.2 The elliptical distribution

An elliptical lift distribution has the form

$$\Gamma(\eta) = \Gamma_0 \sqrt{1 - \eta^2}$$

so the lift per unit span decreases smoothly from the root to the tip, tracing half an ellipse. It produces a uniform downwash along the whole span, and it gives the minimum possible induced drag for a given span and a given total lift,

$$C_{D,i} = C_L^2 / (\pi AR)$$

where AR is the aspect ratio, which corresponds to a span efficiency factor of exactly one. The even downwash contributes to the efficiency and the reduced drag of the wing, and wings designed for this distribution tend to perform well. The principle is famously associated with the elliptical wing of the Supermarine Spitfire, which contributed to its exceptional performance in the Second World War. Achieving an exactly elliptical lift distribution in practice is complicated by structural and manufacturing constraints, but approximating it remains a fundamental aim of aerodynamic wing design.

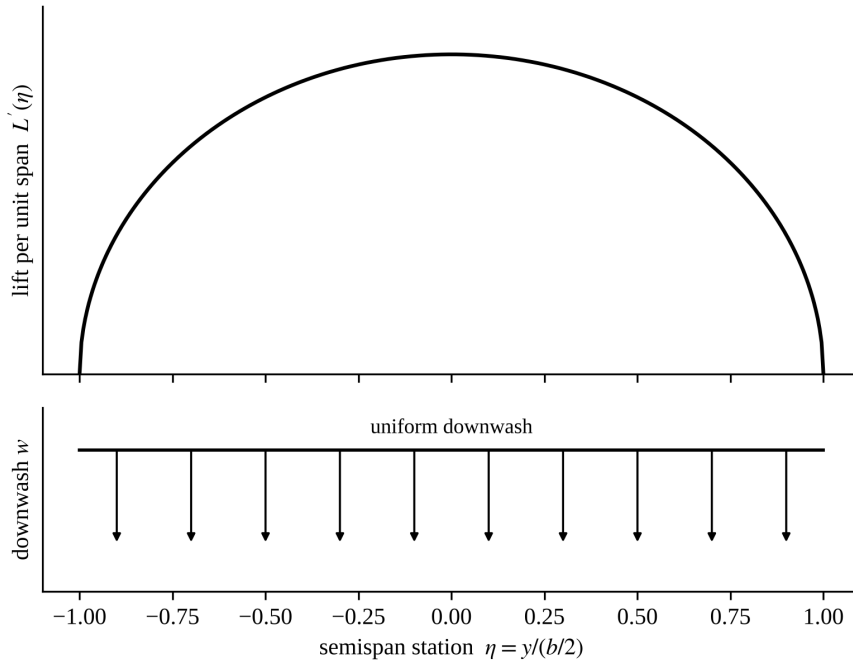


Figure 6. The elliptical lift distribution (top) and its constant downwash (bottom). The even downwash is what gives the minimum induced drag for a given span and total lift.

4.3 The bell-shaped distribution

The elliptical distribution is optimal when the span and total lift are held fixed. Structure, however, depends strongly on the bending moment at the wing root. If the structural weight, represented by the root bending moment, is held constant instead of the span, is there a lift distribution that gives lower induced drag than the elliptical one? It turns out that there is. In 1933 Prandtl published a paper presenting a superior span load to his earlier elliptical distribution, which became known as the bell-shaped lift distribution. It gives the lowest induced drag for a given lift and a given root bending moment. He published it in a German soaring magazine, it was never really translated into English, and it was largely forgotten by most of the aerodynamics world.

A simple analytical form that captures the bell shape is

$$\Gamma(\eta) = \Gamma_0 (1 - \eta^2)^{3/2}$$

which is compared with the elliptical form, which has the exponent 1/2. The larger exponent unloads the tips and concentrates the lift inboard. The consequences for the downwash can be derived exactly. Writing the circulation as a Fourier sine series in the angle θ , where $\eta = \cos \theta$, the elliptical distribution is a single term, Γ proportional to $\sin \theta$. The bell-shaped distribution is proportional to $\sin^3 \theta$, which equals $(3 \sin \theta - \sin 3\theta)/4$. The downwash at each station is the sum over n of $n A_n \sin(n\theta)/\sin \theta$, and for the bell-shaped distribution, normalised to the same total lift, this evaluates to

$$w(\eta) = 2 (1 - 2 \eta^2) \times (\text{the elliptical downwash})$$

The downwash is therefore twice as large as the elliptical value at the centreline, it falls to zero at $\eta = 1/\sqrt{2}$, about 0.71 of the semispan, and it becomes an upwash outboard of that station. This agrees with the statement that the downwash turns into upwash at roughly 70 percent semispan. A direct calculation also shows that, for the same span and the same lift, this particular shape has an induced drag 4/3 times that of the elliptical wing. The benefit appears only when the comparison is made at

equal root bending moment, because the unloaded tips lower the bending moment and so allow a longer span. Prandtl's published result for his optimum is usually quoted as a span about 22 percent greater with about 11 percent less induced drag. For the simple $\sin^3$ shape used here, the same direct calculation gives about 25 percent more span and about 15 percent less induced drag. The exact figures depend on how the structural constraint is defined, but the qualitative conclusion, that a tip-unloaded, bell-shaped distribution beats the elliptical one when structure is accounted for, is the same.

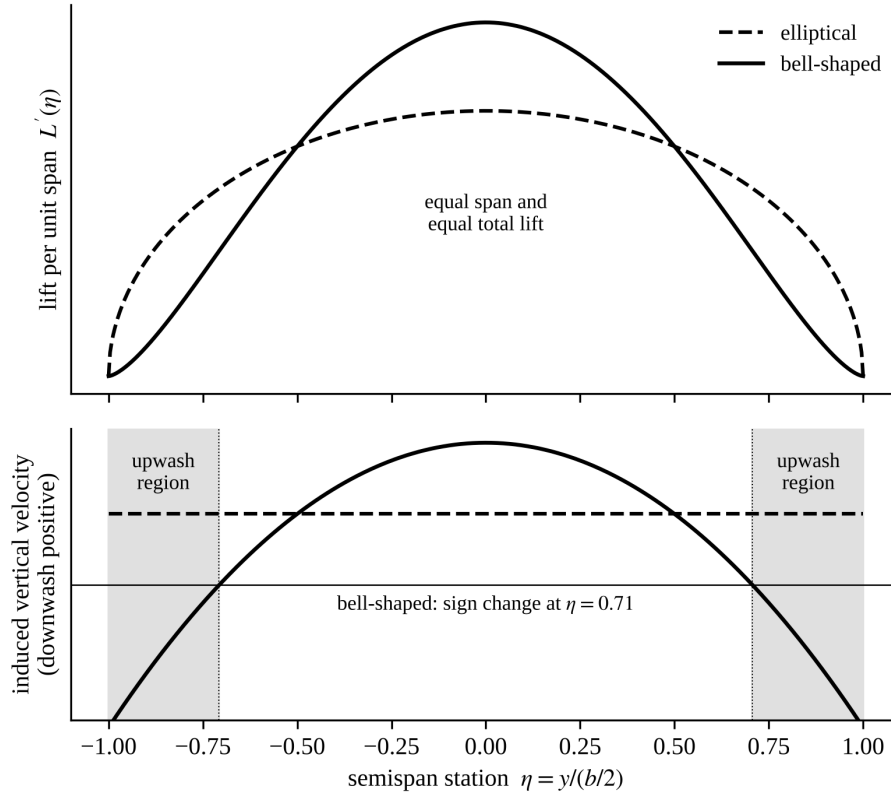


Figure 7. Elliptical and bell-shaped span loads of equal span and total lift (top) and the corresponding induced vertical velocity (bottom). For the bell-shaped load the induced velocity changes sign at 0.71 of the semispan, so the outer wing sits in upwash.

4.4 Induced thrust at the tips

In the bell-shaped distribution the trailing vortices are deliberately located inside the span rather than at the tips. As a result, the tips experience an upwash. Because the local flow is tilted upward, the local lift vector, which is perpendicular to the local flow, leans forward, and a component of it points along the flight direction. This is an induced thrust at the wing tip. Compare this with the elliptical case, in which the aim is an even downwash behind the wing and the lift vector leans rearward everywhere.

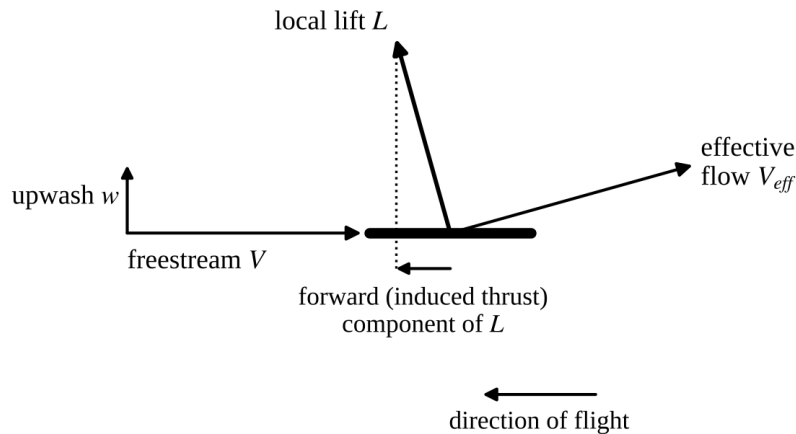


Figure 8. Velocity triangle at a wing-tip section in upwash. The effective flow is inclined upward, so the lift, which is perpendicular to it, is tilted forward and has a forward component that acts as thrust.

4.5 Proverse yaw and roll control

Consider a roll manoeuvre. On a conventional wing, the wing that is rising has its aileron deflected downward, and this surface sits in downwash, which increases its drag. The extra drag on the rising wing yaws the aircraft opposite to the direction of roll, a phenomenon called adverse yaw. With a bell-shaped distribution, the deflected-down elevon or aileron sits in upwash instead. Its drag is reduced, and the forward-leaning lift vector gives a net forward force, so the rising wing is pulled forward and the aircraft yaws in the same direction as the roll. This is proverse yaw: rolling to the right also yaws the aircraft to the right.

Because the tips are unloaded in this distribution, roll control is also retained during and after a stall event, which greatly reduces spin tendencies. The beneficial yaw produced by the induced thrust at the tips was demonstrated experimentally in 2015 by Albion Bowers and co-workers. It eliminates the need for a vertical tail to coordinate turns, and it is exactly why birds do not have vertical tails. Historically, the Horten flying wings used a highly tapered wing with a bell-shaped lift distribution over the span, which produced a small download at the tips. This reduced adverse yaw and allowed them to fly without a vertical tail, but achieving the distribution required extensive washout, sometimes up to 8 degrees.

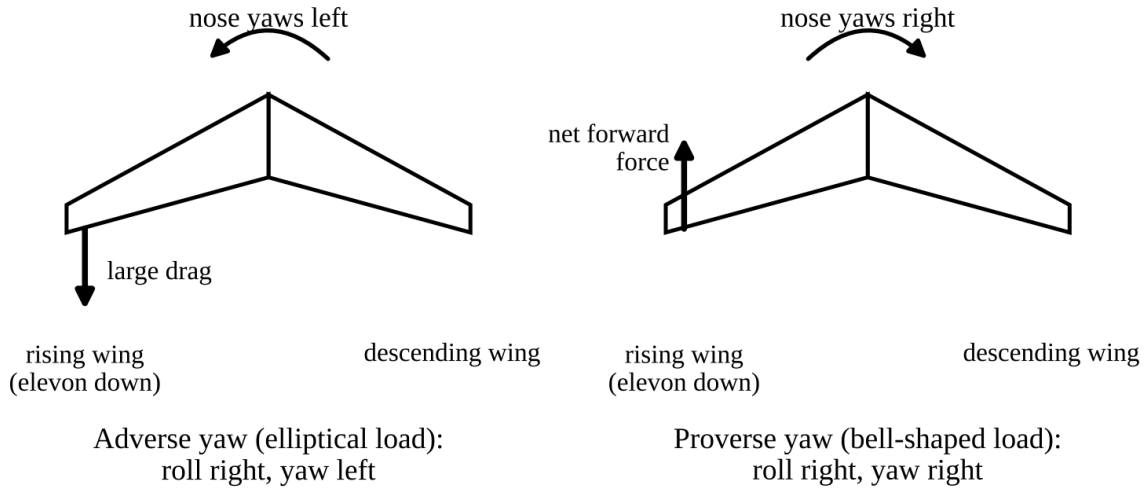


Figure 9. Adverse yaw (left) and proverse yaw (right) during a roll to the right. With an elliptical load, the downward-deflected control surface on the rising wing creates extra drag and yaws the nose left. With a bell-shaped load, the same surface sits in upwash, produces a net forward force, and yaws the nose right.

5. Geometric and Aerodynamic Design

5.1 Wing twist and washout

Washout is a reduction of local incidence from the root to the tip, that is, a negative twist angle ϵ_t at the tip. In effect it places a little tail at the end of the wing. The most important effect, for a swept wing, is on pitch stability. Because of the sweep, the tip is far aft of the centre of gravity and has a large moment arm. At the trimmed condition a washed-out tip carries little lift, or a download, which is what supplies the positive $C_{m,ac}$ required by the trim equation of Section 3.3. As the angle of attack increases, for example in a gust, the lift at the tip becomes increasingly positive compared with the lift at the root, and the lift acting far aft of the centre of gravity pushes the nose back down, just as a traditional horizontal stabiliser does. With a combination of sweep and twist, the lift-producing tips at the root stall angle have a large moment arm about the centre of gravity, and this gives a strong stall recovery.

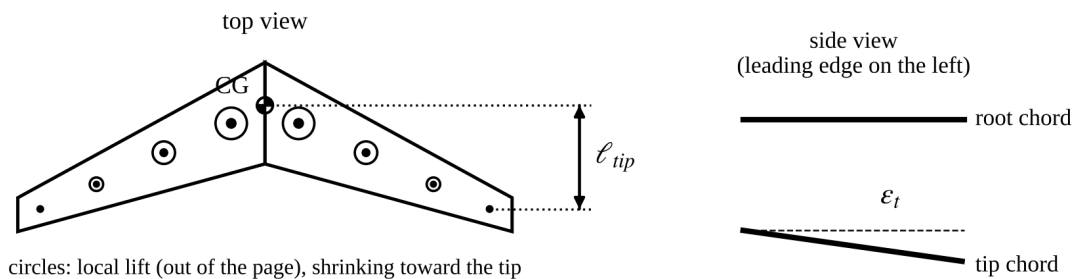


Figure 10. Left: a swept wing with the local lift shrinking toward the tip, whose long moment arm ℓ_{tip} about the CG makes it act like a tail. Right: the tip chord is set at a negative incidence ϵ_t relative to the root chord (side view, exaggerated).

Washout has several further benefits. By unloading the tip, it reduces the strength of the wing-tip vortex and the associated induced drag, at the cost of a reduction in lift across the span and so in the maximum lift coefficient. It also improves lateral stability, the yawing moment, which is especially

important at low Reynolds numbers and high angles of attack. Finally, it delays the onset of stall at the wing tips. Swept wings tend to stall at the tips first, because of the lateral (spanwise) boundary-layer flow, and washout counteracts this. Aileron or elevon control authority can then be maintained while the root of the wing is stalled at high angles of attack. In summary, twist reduces the maximum section lift and the lift distribution across the wing, decreases the overall drag, improves yaw stability and delays tip stall.

5.2 Producing the twist: geometric and aerodynamic washout

There are two ways of producing the washout effect. In geometric twist, the root and the tip are the same airfoil, but the tip is set at an incidence several degrees below that of the root. This is simple, but it produces significantly more drag than the alternative. In aerodynamic twist, the effect of geometric washout is replicated by using a different airfoil at the tip from that at the root, with the tip airfoil having little or no negative twist. If airfoils with different zero-lift angles are chosen, the amount of geometric twist needed can be reduced. The difference between the zero-lift angles of the two airfoils is called the aerodynamic twist,

$$\epsilon_{\text{aero}} = \alpha_{L=0,\text{tip}} - \alpha_{L=0,\text{root}}$$

and the zero-lift angles are found from the airfoil polars. The effective twist seen by the lift distribution is the sum of the geometric and aerodynamic parts,

$$\epsilon_{\text{eff}} = \epsilon_{\text{geo}} + \epsilon_{\text{aero}}$$

A small negative, or even positive, pitching-moment coefficient in the tip airfoil further reduces the required geometric twist and improves the overall performance of the wing.

5.3 Sizing the twist for stability

A well-tested closed-form method for determining the total twist needed for stable flight early in the design process was derived by a Dr. Pegan, as presented in the source video. The formula itself is not reproduced in the source transcript and is therefore not restated here. The method gives only the total twist required for longitudinal stability of a monolithic wing with straight leading and trailing edges and a predetermined static margin, and it has been shown in practice to be accurate for sweep angles up to about 30 degrees, for tapered or constant-chord wings. The inputs in the design spreadsheet described in the source are the weight, the flight speed, the sweep, the taper and the root and tip airfoil polars, specifically their zero-lift angles. The output is the geometric twist needed for a basic flying wing at a chosen lift coefficient and static margin.

The origin of the requirement can be understood from the trim equation of Section 3.3. The pitching moment of the complete wing about its aerodynamic centre has a part from the airfoil sections and a part from the twist,

$$C_{m,\text{ac},\text{wing}} = C_{m,\text{ac},\text{sections}} + K \epsilon_t$$

where K is a coefficient, negative in sign, that depends on sweep, taper and aspect ratio and that can be obtained from a lifting-line or vortex-lattice analysis. It grows in magnitude with sweep, because a larger sweep moves the tip further aft. Requiring $C_{m,\text{ac},\text{wing}} = C_{L,\text{design}} \text{ SM}$ and solving for ϵ_t gives the washout needed for the chosen static margin. In practice the airfoil moments, sweep, taper and twist are traded against one another, and a tip airfoil with a positive moment reduces the geometric twist needed.

5.4 Taper ratio

The taper ratio λ is the tip chord divided by the root chord. Many flying wings are tapered, often strongly, and the primary aerodynamic advantage is a reduction of induced drag. Tapered wings distribute lift more evenly across their span, which reduces the intensity of the wing-tip vortices. The strength of a tip vortex is directly proportional to the pressure difference between the high-pressure and low-pressure sides of the wing. The wing tips are aerodynamically the least efficient part of the wing, generating the most drag for the least lift, especially without winglets or end plates, and tapering biases the lift and drag towards the root.

There are structural benefits as well. Tapering shifts the lift inboard and reduces the bending moment, so the wing spar can be lighter. The lift-generated bending moment is kept more nearly constant across the span than for an equivalent rectangular wing, at the cost of slightly lower total lift. From a performance standpoint, the leading-edge sweep that often accompanies taper can delay shock-wave formation and reduce wave drag at transonic speeds. The reduced tip loading and tip mass lower the roll moment of inertia, which makes the aircraft more responsive to aileron input, and the taper can create an effective dihedral, which reduces the tendency to roll in response to sideslip.

5.5 Stall behaviour and its remedies

The principal drawback of a low taper ratio is that the tip sections operate at a lower Reynolds number than the root sections, because the Reynolds number is proportional to the local chord. At high angles of attack the tip sections are therefore more likely to stall before the root sections. A stalled tip causes the aircraft to roll and to lose lift on that side at the same time. Because the stalled sections are behind the centre of gravity on a swept-back wing, the loss of tip lift also moves the centre of pressure forward and tends to pitch the nose up, which pushes the angle of attack still higher, and the drooping wing leads to the nose-down attitude that is a classic symptom of entering a spin. The separated tip flow also reduces aileron effectiveness, making it harder for the pilot to lift the drooping wing and prevent the roll, and so to avoid entering the spin.

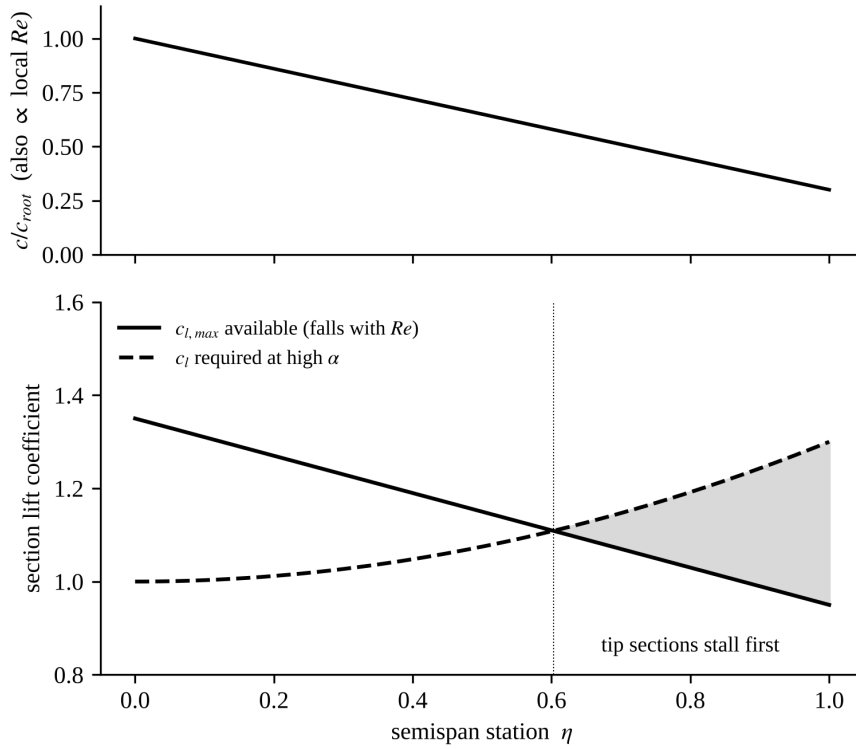


Figure 11. Illustrative spanwise variation of chord (top) and of the required and available section lift coefficients (bottom) for a tapered wing without washout. Because the smaller tip chord runs at a lower Reynolds number, the available $c_{l,max}$ falls toward the tip, and the tip sections reach it first.

Several remedies can be applied. First, the designer can progressively twist the wing so that the tip sections fly at a lower incidence than the root, as described in Section 5.1. Second, the section profile can be changed gradually so that the tip has a different thickness and camber from the inboard sections, which delays the wing incidence at which the tips stall; this is aerodynamic twist again. Third, the designer can introduce stall-inducing devices at the wing root, such as an increased nose profile or stall (trip) strips. Their function is to initiate flow separation on the upper surface of the root sections at high angles of attack, so that the stall starts at the root, which creates a more controlled stall across the wing, helps to prevent entry to a spin, and improves the controllability of the aircraft as it approaches stall.

Tapered wings also have a more complex shape, which increases manufacturing complexity, particularly in sheet metal. Modern composites have mitigated some of these issues, but the shape can still be challenging to make. For most applications a tapered wing is ultimately chosen for its higher performance or load capability despite the more demanding handling at high angles of attack.

6. Summary of Key Relations

The central results of this module are collected here for reference.

$$\begin{aligned}
 SM &= h_{np} - h_{cg} \\
 C_{m,\alpha} &= -C_{L,\alpha} SM \text{ (stable if negative)} \\
 h_{cp} &= h_{ac} - C_{m,ac} / C_L \\
 dh_{cp}/d\alpha &= C_{m,ac} C_{L,\alpha} / C_L^2
 \end{aligned}$$

Trim and stability: $C_{m,ac} = C_{L,design}$ SM

$$C_{D,i} = C_L^2 / (\pi AR e), \text{ with } e = 1 \text{ for the elliptical distribution}$$

$$\text{Bell-shaped load: } \Gamma(\eta) \text{ proportional to } (1 - \eta^2)^{3/2}$$

In words, a tailless wing is stable when its centre of gravity is ahead of its aerodynamic centre, and it can trim at a useful lift coefficient only if it has a positive pitching moment about the aerodynamic centre. That moment is provided by reflexed airfoils, by washout on a swept wing, or by both, at some cost in lift and drag. A bell-shaped lift distribution, produced by heavy washout and taper, lowers the structural weight and induced drag, and it supplies proverse yaw in place of a fin. The accompanying penalty is the risk of tip stall, which is managed with twist, section changes and root stall strips.

7. Review Questions

1. Define the centre of pressure, the aerodynamic centre and the neutral point, and state which two coincide for an isolated airfoil.
2. Starting from the moment about the centre of gravity written in terms of the aerodynamic centre, show that stability requires the centre of gravity to be ahead of the aerodynamic centre.
3. Show that the centre-of-pressure travel is $dh_{cp}/d\alpha = C_{m,ac} C_{L,\alpha} / C_L^2$, and explain its sign for cambered and reflexed airfoils.
4. Why is a positively cambered airfoil balanced at zero angle of attack, with the centre of gravity behind the aerodynamic centre, unstable?
5. Explain why a swept, washed-out wing behaves as though it had a tail. How does the moment arm change with sweep?
6. Compare geometric and aerodynamic twist. Why does aerodynamic twist usually produce less drag?
7. Why can the bell-shaped distribution give lower induced drag than the elliptical one, and under what constraint is the comparison made?
8. Explain how tip upwash produces both induced thrust and proverse yaw, and why this reduces the need for a vertical tail.
9. List the advantages and disadvantages of a strongly tapered flying wing, and describe three design remedies for its stall behaviour.
10. A flying wing has a static margin of 0.05 and a design lift coefficient of 0.6. What pitching-moment coefficient about the aerodynamic centre does the complete wing need in order to trim? (Answer: $C_{m,ac} = 0.05 \times 0.6 = +0.03$.)

8. Topics for the Follow-On Module

The next module will cover static and dynamic stability and the modes that matter most for flying wings (phugoid, short period, Dutch roll, spiral and roll subsidence), a deeper look at stability derivatives and which of them are more and less important for tailless aircraft, and finally the application of all these principles to the design, analysis and test of an actual flying wing.